Problem Set 1, String Phenomenology

May 21, 2008

1 String loop corrections to gauge couplings on D-branes

a) Finish the calculation of moduli-dependent annulus diagram contributions Δ to $g_{\rm YM}^{(1-{\rm loop})}$ we begun in class, by showing that

$$\Delta = \int_0^\infty d\ell \ \Gamma_{\phi,U}(\ell) = -\frac{1}{2}\ln\left|\frac{\vartheta_1(\phi/2\pi,U)}{\eta(U)}\right|^2 + \frac{(\operatorname{Im}\phi)^2}{4\pi\operatorname{Im}U} \tag{1}$$

where ϑ_1 is a Jacobi theta function (Polchinski 7.2.38d), Γ is the Kaluza-Klein sum for branes of separation $\phi = a_1 + Ua_2$ on a spacetime 2-torus of metric G and complex structure U, i.e.

$$a\Gamma_{\phi,U}(\ell) = \vartheta[{}^{\vec{0}}_{0}](\vec{a}/2\pi, 2i\ell G) = \sum_{n_1,n_2} e^{-2\pi\ell n_i G^{ij} n_j} e^{i n_i a_i} \qquad (i = 1,2)$$

with the normalization constant $a = -(2\pi)^2 U_2$, and I have dropped terms that *diverge* when the large-ℓ cutoff Λ is taken to infinity. (These infinite terms cancel between diagrams.) Using results derived in class, this should be fairly easy.

b) Assuming you only know the first term on the RHS of (1), calculate the second term on the RHS by imposing that the gauge coupling should come back to itself if the D-brane goes once around the torus. (*Hint*: Polchinski Vol I, discussion above 7.2.3)

2 Orbifold model building

Put the branes at the point $\phi = \pi$, and set $U = iU_2$ for real U_2. Assuming gauge unification (cf. discussion in class, and Polchinski 16.4.32) and using the formula valid for the orbifold that we derived in the lecture notes,

$$c\Delta = -\frac{14}{5}\left(\ln\left(\frac{M_{\rm s}^2}{M_{\rm GUT}^2}\right) + \frac{32\pi\delta_{\sin^2\theta_{\rm W}}}{28\alpha_{\rm em}(M_Z)}\right) , \tag{2}$$

and experimental input from `pdg.lbl.gov`, then for $M_{\rm s}/M_{\rm GUT} = 20$, $c = -8$, calculate U_2, using at one loop *only* the contribution Δ from problem 1 above. (This is only for simplicity, since it is not the complete answer.) It is enough if you enclose a plot of $\Delta(U_2)$ and estimate the required range(s) for U_2. You have now used measurements to calculate something about the shape of the extra dimensions in which our brane world is embedded! However, even if we would live in an orbifold brane world, name at least two weaknesses with this argument.

3 LHC counting signatures (may leave until next problem set)

We discussed in class that LHC counting signatures from “string” models in the literature are believed to be fairly effective at distinguishing between different models. Here are a few simple questions. Please write concise answers, e.g. 3 sentences per question.

a) List a few difficulties with this procedure for hadron colliders that would be (at least partially) improved with e^+e^- colliders.

b) One of the counting signatures is “number of b-jets”. List at least one reason why this signature is useful (ie. something about the spectrum it is sensitive to), and one way in which it can be misleading.

c) Explain in your own words why, if your string model has light squarks, you might expect a non-vanishing "lepton charge asymmetry" signature at the LHC, and what sign it should be.