String Phenomenology, Part 3, v1.1

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Welcome to the end of the PhD course! I hope you had some fun. Here is a summary of my last lecture, and an attempt to connect it with what you did in the problems. I am sure I will think of improvements to this, so for those of you who might be interested, when I have time next (probably in a few weeks) I'll post a final version and send out an email.

1 The impact of KKLT

In terms of citations, KKLT [1] is the biggest splash in string theory in the last 5 years[1]. One should be very skeptical of attempts to link citation status with actual importance, but to you as a graduate student, this means you should probably be *aware* of KKLT. Whether you choose to *work* on it in the future is hopefully a matter of whether you perceive it is actually important rather than its citation status.

2 GKP: "KKLT v0.9"

Most papers are not ideas that came out of nowhere but have a long and often complicated prehistory. Many of the important things in KKLT were already explicitly stated in the beautiful paper [2] by Giddings, Kachru and Polchinski from 2001 (when I was finishing graduate school). They studied compactification of Type IIB string theory to four dimensions on warped Calabi-Yau manifolds with background 3-form and 5-form fluxes (see Polchinski Ch. 12.1 for notation):

$$\begin{aligned} ds^2_{10} &= e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}dy^m dy^n && (1)\\ G_3 &= F_3 - SH_3 \neq 0 \quad \text{with} \quad \star_6 G_3 = iG_3 && (2)\\ \tilde{F}_5 &\neq 0 && (3)\end{aligned}$$

where $\tilde{g}_{mn}$ is the 6d Calabi-Yau metric. Noone knows an analytical expression for $\tilde{g}_{mn}$, only topological information such as the number of independent complex structure and Kähler moduli (metric deformations), though some *numerical* information about this kind of metrics has recently been extracted in simple cases, like [3]. As discussed before, tadpole cancellation is a crucial constraint on any orientifold model, for example for the D3-brane tadpole constraint: [2]

$$N_{\rm D3} + N_{\rm flux} - \frac{1}{4}N_{\rm O3} = 0\ . \tag{4}$$

because the O3-plane charge is $-1/4$ that of a D3-brane. [3] This compactification leads to an $\mathcal{N}=1$, $D=4$ supergravity coupled to a set of moduli scalars S, T_i, U_α: the complexified dilaton S, the complex structure moduli U_α (cf. our discussion of complex structure U in the torus case), and the Kähler moduli

[1] As of June 2008: typing FIND TOPCITE 1000+ AND DATE AFTER 2002 into SPIRES gives only one string theory paper, KKLT.

[2] Here is a note for those of you who already know some supergravity: We have discussed tadpole conditions from string theory. You can also think of the D3-brane tadpole condition as the Bianchi identity/equation of motion for the associated flux, i.e. $\tilde{F}_5$, integrated over the internal manifold, as in GKP eq. (2.24). The 3-form fluxes give effective D3-brane charge that contributes to this equation because of the 10d Chern-Simons term $\int C_4 \wedge F_3 \wedge H_3$. We then find explicitly $N_{\rm flux} = 1/((2\pi)^4\alpha'^2)\int H_3 \wedge F_3$, and as explained in Polchinski Ch. 13.2, the fluxes are quantized such that this expression is always integer.

[3] In general it is -2^{k-5} for a standard O-plane with k spatial dimenions, see Polchinski Vol II p.143, and here $k=3$. (I say "standard" because there are also "exotic" O-planes, see [4] p. 39.)

T_i (cf. our discussion of the torus area). The K and W functions are (see eq. (23) below)

$$K_{\rm GKP} = K_{\rm tree} = -2\ln\mathcal{V} \quad (\mathcal{V} \text{ is volume, a function of } T_i) \tag{5}$$

$$W_{\rm GKP,start} = \int G_3 \wedge \Omega \quad (G_3 \text{ depends on } S, \text{ and } \Omega \text{ depends on } U_\alpha) \tag{6}$$

where Ω is the holomorphic 3-form of the Calabi-Yau manifold (see Maxim's lectures!) for some given complexified 3-form flux G_3. Don't worry right now if you don't understand (6), it will not be our main focus here. It should be clearly stated that there are many caveats here, one of which is how these equations are affected by warping (a few recent papers with references to earlier work are [5], [6]). In KKLT, warping is an important part of the construction so we *do* need to worry about this, but it is at least possible that in the "LVS" setup I discuss later, warping may be relatively unimportant. The $\mathcal{N}=1$, $D=4$ supergravity scalar potential is (Polchinski (B.2.29))

$$V = e^K (K^{\bar{J}I} D_{\bar{J}}\bar{W} D_I W - 3|W|^2) \qquad I = (S, U_\alpha, T_i) = 1, \ldots, \text{total number of moduli} \tag{7}$$

where the Kähler covariant derivative $D_I W$ with respect to a generic modulus scalar ϕ_I is

$$D_I W = \partial_I W + K_I W \tag{8}$$

and indices on the Kähler potential K mean derivatives, e.g. $K_I = \partial_I K$ and $K^{\bar{J}I} = (\partial_I \partial_{\bar{J}} K)^{-1}$. When you plug (5) into this scalar potential V, you stabilize the S and U_α moduli, meaning the potential V has a minimum at some values of S and U_α, so those moduli will settle down there. We can then plug these values of S and U_α into (6) and write an effective theory for only the Kähler moduli T_i: [4]

$$K_{\rm GKP} = K_{\rm tree} = -2\ln\mathcal{V} \qquad (\mathcal{V} \text{ is a function of the real part of } T_i) \tag{9}$$

$$W_{\rm GKP} = W_0 \tag{10}$$

for a constant W_0. Now, it turns out we will not stabilize the T moduli this way. This is because the tree-level Kähler potential (9) happens to satisfy

$$K^{\bar{i}j} K_i K_{\bar{j}} = 3 \qquad (i \text{ runs over Kähler moduli } T_i \text{ only}) \tag{11}$$

so the term with $D_{T_i} W$ cancels the $-3|W|^2$ in (7) (can you see this?). In other words, (9) and (10) give a four-dimensional model in which T_i are not stabilized, so no particular scale is generated. In appendix C.1 and C.2 of [7], we spent a few pages giving a detailed derivation of this fairly well-known statement.

These supergravity models were discovered by Cremmer et al in 1983 [8], and it was considered interesting that they can have zero cosmological constant $V_0 = 0$ (V_0 is the value of V at the minimum) even after spontaneous supersymmetry breaking. [5] However, after supersymmetry breaking there is no symmetry preventing loop corrections or string corrections from generating a large cosmological constant, so this property seems more of a curiosity rather than a solution of the cosmological constant problem. As Weinberg says in his book, [p.355] "there is no known principle that would require [no-scale]".

There *was* no known principle. One of GKP's contributions was to emphasize (section 2.2.4) that for orientifold models, tadpole cancellation plus supersymmetry enforces no-scale, so no-scale is more than a curious coincidence: it is generic at supergravity tree-level in supersymmetric orientifolds. However, they phrased it more as a problem than a virtue; no-scale by definition means we cannot stabilize T_i at supergravity tree-level. For moduli stabilization, this is bad news: if you thought you could do phenomenology in partially stabilized models, see the cautionary tale on p.4. The good news is that (as GKP also emphasized) the no-scale structure would be broken by every possible correction:

[4] This "two-step stabilization" is not obviously consistent here, because when we later stabilize T, that might "un-stabilize" S and U. In the cases we'll look at later, it will be consistent.

[5] For the experts, the way to see that you don't generate a cosmological constant when you have a constant superpotential W_0 is that the F-terms $F^T \neq 0$ due to the $\partial_T K$ term in $D_T W$, but this cancels in V. If we still keep $F^U = F^S = 0$, we don't generate a cosmological constant, but supersymmetry is broken by $F^T \propto D_T W \neq 0$.

a1) nonperturbative gauge theory effects on D-branes

a2) nonperturbative effects in g_s (D-brane instantons)

b1) perturbative string α' corrections

b2) perturbative string g_s corrections

so the recipe to stabilize T_i would be to consider one of these effects. (Of course, generating a potential for T_i is not enough; one has to find a minimum, and show the minimum is at reasonably large volume to neglect α' corrections.) Let's call this the "GKP list", though noone calls it that. What KKLT did was to consider case a) (not really specifying whether a1 or a2). We will come to cases b1) and b2) later.

3 Stabilizing Kähler moduli

So, one of the two things in KKLT was to add to (9) and (10) a nonperturbative superpotential, as per option a1) and a2) in the GKP list:

$$K_{\text{KKLT}} = K_{\text{tree}} = -2\ln\mathcal{V} \tag{12}$$

$$W_{\text{KKLT}} = W_0 + W_{\text{np}} \quad \text{where} \quad W_{\text{np}} = \sum_i A_i e^{-a_i T_i} \tag{13}$$

and the sum runs over cycles *for which such a* W_{np} *is generated* by these nonperturbative effects. There are examples both where *all* cycles get a W_{np} (i in the sum above runs from 1 to the number of Kähler moduli) and ones where only some get a W_{np} (the i sum is restricted). On the good side, this means that at least we have *some* examples where we achieved our goal, which was to stabilize *all* moduli. But it also means we have to check on a case by case basis whether we stabilize all of them this way.

The other of the two main things in the KKLT paper was supersymmetry breaking.

3.1 Supersymmetry breaking by "uplift", and problems

"Uplift" means we initially stabilize in a supersymmetric AdS vacuum (negative cosmological constant $V_0 = \Lambda_{\text{AdS}} < 0$ in our external four dimensions), then add something that breaks supersymmetry, like anti-D3-branes, or non-anti-selfdual D7-brane flux, to "lift up" the vacuum energy to zero $V_0 = 0$ (Minkowski) or positive $V_0 = \Lambda_{\text{dS}} > 0$ (de Sitter). In these notes I will focus on zero cosmological constant. [6]

Either way, it is quite problematic to uplift the way it was envisaged in KKLT. For a more recent approach, see [9] and references therein. The statement in the next section is that the "uplift mechanism" is not supposed to affect the calculations much, and that most of the physics comes from the supersymmetric theory. This is a very strong statement, and to a purist it sounds like saying that my Mac Pro will be totally OK if the entire AlbaNova building collapses. If the statement turns out to be false, then we need to retreat to something more along the lines of [9]. But thus far, there have been quite a few surprises, so we should not guess too much and calculate instead.

To summarize, although the consistency of the supersymmetry breaking approach in the KKLT paper itself was at best unclear, KKLT did start a wave of considering metastable nonsupersymmetric solutions, some of which appear to be more consistent. In fact, they inspired searching for metastable supersymmetry breaking minima in non-gravitational gauge theories used in particle physics, as in ISS [10][7], which is quite an interesting development in itself.

[6] Comment: even though positive Λ can appear to be the entire point of KKLT (the title of the paper is "de Sitter vacua in string theory"), the goal of stabilizing all moduli is independent of the physical motivation of studying de Sitter. In fact, in KKLT the absolute value of the original Λ_{AdS} is much, much bigger than the *observed* amount of dark energy, so when we uplift to positive Λ, we have to go just a tiny, tiny bit above zero if we want to model the observed dark energy, so the value is put in by hand. This means that if the goal is to use the KKLT recipe to stabilize all moduli, we can equally well consider uplifting to Minkowski space, which simplifies a few things (see sec. 5)

[7] Curiously, this paper does not cite KKLT, although some of the authors have informally indicated some inspiration from KKLT.

3.2 A cautionary tale

We could take W from (6) above and perform the intermediate-stage calculation

$$\mathcal{L}_{\rm soft} = \ldots \qquad \text{(intermediate)} \tag{14}$$

and do MSSM-style phenomenology with this. But when we add $W_{\rm np}$ to stabilize T, supersymmetry is actually preserved in the AdS minimum. This means that after stabilization the result is changed to

$$\mathcal{L}_{\rm soft} = 0 \quad ! \qquad \text{(final)} \tag{15}$$

All supersymmetry breaking then comes from the "uplift", which does not depend on details of (14). So in this example, our "partially stabilized" phenomenology of $\mathcal{L}_{\rm soft}$ gives very little information about physics after complete stabilization, which is all we care about for phenomenology. The lesson learned is that "partial stabilization" can be very misleading.

3.3 KKLT didn't invent $W_{\rm np}$

The possibility in principle of considering nonperturbative superpotentials to stabilize Kähler moduli as was obvious to many people before KKLT (it's in the "GKP list" for example), but the possibility of getting metastable vacua with extremely long lifetime was less obvious. The other possibilities b1) and b2) were only afforded a comment in KKLT, but various extensions of KKLT has explored them, as in the LVS variant we turn to now.

4 LVS: "KKLT v1.1"

In the same way, the *idea* of considering perturbative corrections to the Kähler potential as in b1) on the "GKP list" in principle was obvious to most people, but the *effect* of doing so was not obvious, and in fact is still somewhat mysterious. It was discovered in [11] that if we include the α' correction calculated in [12] to the Kähler potential,

$$\Delta K_{\alpha'} = -\frac{\xi S_1^{3/2}}{\mathcal{V}} \qquad \xi = -\zeta(3)\chi/2(2\pi)^3 \tag{16}$$

where $S_1 = \mathrm{Re}\, S$ and χ is the Euler number of the compactification manifold, *there seem to exist new nonsupersymmetric AdS minima at very large volume.* This is counterintuitive because typically if you try to balance a tree-level term against an α' correction, you can only stabilize at the string scale. This is an important objection, so let me give some more detail. Let's say one 4-cycle volume controls the overall scale of the compactification manifold, and that this is real part of one of the Kähler moduli $\tau = \mathrm{Re}\, T$. The experience with "Kähler stabilized" models where you stabilize by adding something like (16) is that τ ends up of order $\ell_{\rm s}^4$, that is, order one in string units ($\ell_{\rm s} := 1$), i.e. very small rather than very large. If so, we have a challenge: by dimensional analysis we might expect α' corrections to be suppressed as $(\alpha'/\sqrt{\tau})^n$ for powers n, so if $\tau \sim (\alpha')^2$, we would need to consider an infinite number of such α' corrections.[8] This is the "truncation problem" usually associated with α' corrections. (An interesting 1985 discussion of this for the heterotic string is [13].). Here's a toy example:

[8] As was astutely observed during my talk, it doesn't necessarily mean we must consider an *infinite* number, although it does in my toy model. It is unlikely, though, that we would need to consider only leading order. Going far beyond leading order can be OK in gauge theory like QCD where people spent a lot of effort computing e.g. the 5-loop β function, but in string theory, the *leading* corrections are barely known, so considering e.g. next-to-next-to-leading order in α' would probably require another decade worth of effort. There is no problem with that in principle, but for the purpose of this discussion, let's define "an infinite number of corrections" as "far beyond what anyone has computed or is planning to compute".

$$\text{toy model } V: \quad V(\tau) = e^{-\tau}(1+\epsilon\tau+\mathcal{O}((\epsilon\tau)^2) \overset{\text{truncate}}{\longrightarrow} e^{-\tau}(1+\epsilon\tau) \tag{17}$$

$$V'(\tau) = e^{-\tau}(-1-\epsilon\tau+\epsilon) \overset{!}{=} 0 \quad \Rightarrow \epsilon = (1-\tau)^{-1} \sim -1/\tau \quad \Rightarrow |\epsilon\tau| \gtrsim 1 \tag{18}$$

$$\therefore \text{ inconsistent truncation in (17)}$$

But in LVS, as the name suggests, we find that at least with the current state of the art, it appears that one can find minima with volumes as big as $\mathcal{V} = 10^{15}\ell_s^6$, corresponding to τ of order $10^{10}\ell_s^4$. Let's see how this appears to sidestep the truncation problem, and why I'm writing "appears to".

$$K_{\text{LVS}} = K_{\text{tree}} + \Delta K_{\alpha'} \tag{19}$$

$$W_{\text{LVS}} = W_{\text{KKLT}} = W_0 + W_{\text{np}} \tag{20}$$

We now assume that the volume $\mathcal{V}$ is of special form: some of the Kähler moduli, T_b, appear with a plus sign and some T_s in a separate term with a minus sign (remember $\tau = \text{Re}\, T$):

$$\mathcal{V} = \tau_{\text{b}}^{3/2} - f(\tau_{\text{s}}) \tag{21}$$

This is called "Swiss cheese" Calabi-Yau. Plugging (19) and (20) into (7), we find in the large volume limit $\mathcal{V} \to \infty$

$$V_{\text{LVS}} = e^{K_U}\left(\frac{\lambda\sqrt{\tau_{\text{s}}}e^{-2a\tau_{\text{s}}}}{\mathcal{V}} - \frac{\mu}{\mathcal{V}^2}\tau_{\text{s}}e^{-a\tau_{\text{s}}} + \frac{\nu}{\mathcal{V}^3}\right) \tag{22}$$

where λ, μ and ν are calculable constants. Despite appearances, these terms are all of the same order in $\mathcal{V}$; this is because $e^{-a\tau_{\text{s}}} \sim 1/\mathcal{V}$. The third term is new and causes a big change in the minimum structure at large volume. This is the large volume scenario, or LVS.

There are a number of uncertainties about this, that are listed on p. 30 of [7] so I won't repeat them here, but rather take the optimistic point of view that they will be addressed in the future.

5 LVS string phenomenology

Considering that LVS has only existed a few years, it is surprising how much phenomenology has already been done with it. Still, there is no comparison with how much phenomenology has been done in say MSSM-3 models (thousands of papers), so there is certainly room for more work in this direction.

One "drastic" example is [14]. It is drastic in the same sense as Kane et al. In this analysis, however, there is not much phenomenological difference from the MSSM-3 type models, the main difference being that the spectrum is a little more "compressed" due to the lower scale of new physics (10^{11} GeV in LVS versus 10^{16} GeV in MSSM-3), so the renormalization group (RG) evolution from high to low scale has less distance over which to pull the $\mathcal{L}_{\text{eff}}$ parameters apart. If this was the full story, there would not be that much of a reason to study LVS phenomenology further. However, this strong similarity with MSSM-3 models is almost built in from the start in the particular analysis in [14], and there are many reasons to expect that this statement will not survive further work, e.g. the MSSM itself has not been properly embedded here. The most recent work on this is from a few days ago [15].

Another issue left completely open is the cosmological constant problem. It can actually affect observables in principle, since V_0 appears in the soft terms. But in practice, it is argued that at least if we plug in the observed value of Λ for V_0, the contribution is numerically insignificant. However, to really answer this question we would need to know the cosmological constant at the string scale, where we originally calculated the soft terms, for which we would need to know the RG running of the cosmological constant, and this is a problem plagued with ambiguities. There are a few ideas how to possibly address this in LVS, but nothing really concrete so far. This is why I said above that considering Minkowski simplifies things, even though we ultimately might want to consider de Sitter. The reason we might want to is, as you probably know, that people in my group in Stockholm (and others!) have observed dark energy and a positive cosmological constant is one way to model that. But, if you *only* want to do particle physics (say LHC), it is not clear to me why the observed dark energy will become important *right now*, and not have caused some problem in particle experiments say 20 years ago. It would be interesting if that was the case! But here, let's consider Minkowski for the external dimensions.

The most recent analysis is quite recent, Allanach et al [16].

6 The present: LVS and the string effective action

Before, we "only" considered supergravity and scalar moduli fields, i.e. closed string fields. Now we need to consider also open string fields, that will give rise to the supersymmetric version of the standard model. (If you tend to forget which fields in the low-energy effective theory come from closed strings and which come from open strings, review Polchinski Ch. 4, especially p.134-137). To write down the complete effective action it is useful to consider the bosonic action first, then write down the fermionic terms using supersymmetry. Here, we will not get to that at all, but you should keep in mind that if you understand the action for an open string scalar boson ϕ you also know it for its superpartner ψ. The bosonic part of the most general $\mathcal{N}=1$ supergravity + gauge theory effective action with at most two derivatives is $S_{\text{eff}} = \int d^4x\sqrt{-g}\,\mathcal{L}_{\text{eff}}$, where (Polchinski eq. (B.2.28))

$$\mathcal{L}_{\text{eff}} = \frac{1}{2\kappa^2}R - K_{\phi_i\bar{\phi}_{\bar{j}}}\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{j}} - \frac{1}{g^2(\phi)}\text{tr}_a\, F^2 - V(\phi) + \text{corrections} \tag{23}$$

where the nonabelian kinetic term $\text{tr}_a\, F^2$ might be more familiar to you in the coordinates $A = A^a t^a$, where $\text{tr}_a\, F^2 = F^a_{\mu\nu}F^{a\mu\nu}$, and where the ϕ_i are *generic* scalars (including both open and closed string scalars), for which

$$K_{\phi_i\bar{\phi}_{\bar{j}}} = \partial_{\phi^i}\partial_{\bar{\phi}^{\bar{j}}}K \quad \text{and} \quad \frac{1}{g^2(\phi)} = \text{Re}f(\phi) \tag{24}$$

and V is given by (7). One point of expressing the effective action in terms of K, W and f is that this is an efficient way to encode the special properties of an $\mathcal{N}=1$ supersymmetric theory: The functions $f(\phi)$, $W(\phi)$ are holomorphic, and $K_{\phi^i\bar{\phi}^{\bar{j}}}$ is Kähler. (To see how restrictive $\mathcal{N}=1$ is, just compare a Kähler metric, which is determined by one function K, to an arbitrary field space metric $G_{\phi^i\bar{\phi}^{\bar{j}}}$. Another example is that we saw in the MSSM that holomorphy of W forces us to have at least two Higgs fields.). So, if we know K, W and f, we know most of the things we need to know about the $\mathcal{N}=1$ effective action[9]. The "+corrections" in eq. (23) come in two kinds in string theory that have no direct analog in point-particle theories: α' corrections (nonzero string length) and g_s corrections (nonzero string splitting probability). One can first wonder if there are other α' corrections in LVS than the one they considered. There are, but it was argued that they are subleading at large volume; the one they include is topological and so persists at large volume. For potential loopholes in this statement, see section 6 in [7].

There is only one qualitatively new kind of correction left to consider: g_{s} corrections. If g_{s} corrections turn out to make a big difference, it is again at first sight likely that we will end up with an inconsistent truncation. For the same reason as before, the impression as first sight may be misleading, and it is interesting to investigate what actually happens. The gauge kinetic function f and the Kähler potential K are known to receive perturbative corrections, so let's focus on those. The superpotential W is not supposed to receive corrections in perturbation theory (see e.g. Weinberg's book), but here it does in a certain sense, that we discuss at the end.

[9] There are also D-terms and Fayet-Iliopoulos terms, but let's leave them aside for now. Also, if you are used to renormalizable scalar field theories, the nontrivial Kähler metric can look unfamiliar to you; it produces derivative couplings like $\phi^2(\partial\phi)^2$ which looks nonrenormalizable by counting powers of fields and derivatives (if the mass dimension of ϕ is the canonical $[\phi]=1$, that coupling is dimension 6). The logic of keeping such terms is explained in Polchinski, Appendix B under "Higher corrections and supergravity". In any case, gravity is not renormalizable by power counting either.

6.1 Toroidal orientifolds: $\mathcal{N}=2$ sectors

The different contributions are classified into "$\mathcal{N}=2$ sectors" (strings that are stuck at orbifold fixed points only in some directions), and "$\mathcal{N}=1$ sectors" (strings that are completely stuck). Watch out not to confuse this terminology with the total supersymmetry of the effective theory: even an orientifold model with only "$\mathcal{N}=2$ sectors" (such as the $\mathbb{T}^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ orientifold that you might keep in mind as an example in this section), can produce an $\mathcal{N}=1$ supersymmetric effective field theory, which is what we have here.

Let's now consider the "+corrections" in eq. (23) in turn.

A. Corrections to f on D7-branes

$$\left(\frac{1}{g^2}\right)=\left(\frac{1}{g^2}\right)^{\text{tree}}+\left(\frac{1}{g^2}\right)^{1-\text{loop}}+\ldots \tag{25}$$

As you know from the first lecture, to find the string loop correction to the gauge coupling on D-branes of any dimensionality, we can calculate the 2-point function of massless open-string vectors A_μ. We obtained a result that had terms like $(p_1\cdot p_2)$. One technical difficulty is the fact that *any* massless 2-point function that is proportional to $p_1\cdot p_2$ is naively zero by simple Lorentz kinematics. This is because $p_1\cdot p_2=0$ by on-shell-ness and momentum conservation (check this!), even though the end result can be nonzero due to a pole $1/(p_1\cdot p_2)$ that arises from the integral over worldsheet moduli (for example, the integral over the distance ν between the two vertex operators). In other words, the apparent vanishing of the 2-point function can be a 0/0 limit problem. As was explained in a beautiful paper by Minahan in 1987 [17] we can relax momentum conservation, extract the finite contribution to this amplitude, then impose momentum conservation again. We cannot relax the on-shell (BRST) condition, since this would allow unphysical states to propagate. Performing the $\int d\ell$ integral over the Kaluza-Klein or winding sum $\Gamma_{\phi,U}(\ell)$ gives rise to the string version of the Georgi-Quinn-Weinberg equations: (cf. Polchinski 16.4.32)

$$\left(\frac{1}{g^2}\right)^{1-\text{loop}} = \beta^{\mathcal{N}=2}\ln\frac{M_{\text{string}}}{\mu}+\Delta(\phi,U) \tag{26}$$

where μ is the RG scale, $\beta^{\mathcal{N}=2}$ is the RG β function for the $\mathcal{N}=2$ part of the theory and

$$\Delta(\phi,U)=-\frac{1}{2}\ln\left|\frac{\vartheta_1(\phi/2\pi,U)}{\eta(U)}\right|^2+\frac{(\mathrm{Im}\phi)^2}{4\pi\,\mathrm{Im}U} \tag{27}$$

is called the *threshold correction*[10]. Here ϕ means D-brane scalars and U is the complex structure moduli of the relevant torus on which the D-branes are moving (for example, in the D3-D7 system, with all branes space-filling, ϕ can be D3-brane scalars parametrizing motion on the torus transverse to the D7-branes, then we denote the complex structure of that torus by U). Since $1/g^2=\mathrm{Re}f$, and $\ln|z|^2=\ln z+\ln\bar{z}=2\,\mathrm{Re}\ln z$, we have

$$f^{1-\text{loop}}=-2\ln\vartheta_1(\phi/2\pi,U)+\ldots \tag{28}$$

Now, if we have an asymptotically free (negative β function) gauge theory on the D7-branes, we can have gaugino condensation and generate a W_{np}, just as in KKLT. Two comments: first, in the orientifold, the matter content is strongly restricted, so we need to check whether the theory is *actually* asymptotically free so W is indeed generated! (For example, fermions give positive contributions to the β function, see e.g P& S p. 531). Second, although the superpotential W itself receives no perturbative corrections, it can receive one-loop corrections by correcting f! In fact

$$W_{\text{np}}=Ae^{-af}=Ae^{-a(f^{\text{tree}}+f^{1-\text{loop}}+\ldots)}\overset{(28)}{=}\underbrace{A\cdot(\vartheta_1(\phi/2\pi,U)^{2a}\cdots)}_{\tilde{A}(\phi,U)}e^{-a(f^{\text{tree}}+\ldots)} \tag{29}$$

[10] This is because they come from massive states that become relevant as you approach some mass threshold.

so we can think of the calculation of the one-loop correction to f as a calculation of the moduli dependence of the forefactor A in the nonperturbative superpotential. Note that for D7-brane gaugino condensation, $a \sim 1/N_{\rm D7}$ meaning $2a$ is generally not integer, so there is some interesting branch structure as we move the D-branes around (i.e. change ϕ). Incidentally, if the field theory were truly $\mathcal{N}=2$ supersymmetric, there would be no loop corrections to f beyond one loop, so the result would hold to all orders of perturbation theory. Here this is of limited importance since we are interested in $\mathcal{N}=1$ theories.

B. Corrections to K

The direct calculation of Kähler corrections is a little more involved. One complication is that we are not just interested in the Kähler *metric* (e.g. $K^{1-\text{loop}}_{T\bar{T}}$ which is what we get from these string calculations, analogously to gauge coupling corrections) but the Kähler *potential* as a function of all moduli $K^{1-\text{loop}}(S,\bar{S},T,\bar{T},U,\bar{U},\phi,\bar{\phi})$, from which you obtain the Kähler metric by partial differentiation. (For example, the Kähler potential K appears in (7).) In other words, we have to "integrate" the various Kähler metric corrections to find a single function $K^{1-\text{loop}}$, and there are "integrability conditions". Also, the vertex operators for scalars depend more on details of the internal manifold (they are "polarized in the internal directions") than they do for vectors.

The end result is that we find this correction to the Kähler potential [11]

$$\Delta K^{1-\text{loop}} = \frac{1}{128\pi^6}\frac{1}{S_1 T_1^3}\sum_{i\in\text{stacks}} \mathcal{E}_2(\phi_i, U) \tag{30}$$

where $\mathcal{E}_2(\phi_i,U)$ is a sum over images and surfaces (eq. (2.68) of [18]) of the generalized nonholomorphic Eisenstein series

$$E_2(\phi,U) = \sum_{m,n}{}' \frac{U_2^2}{|n-mU|^4}\exp\left[2\pi i\frac{\bar{\phi}(n-mU)-\phi(n+m\bar{U})}{U+\bar{U}}\right] \tag{31}$$

Notice that the exponent is just $e^{2\pi i n_i a_i}$ written in complex coordinates, where $\phi = a_1 + U a_2$ and $a_i \in \mathbb{R}$, as you wrote in Problem 1.1.

C. Consistency check

For the f and K corrections to be consistent, there is a relation that needs to be satisfied. At least at points of enhanced gauge symmetry $\phi_i = 0$, there is an "$\mathcal{N}=2$ relation" between f and K, at each order in the string coupling. This is similar to the heterotic case studied in 1995 [19]. From there we find

$$\mathrm{Re}\, f^{1-\text{loop}} = \frac{1}{2\pi i}e^{-K}K^{1-\text{loop}}_{\phi\bar{\phi}} \tag{32}$$

which reduces, if we plug in results from A and B above, to

$$\partial_\phi\partial_{\bar{\phi}}E_2(\phi,U) = -\frac{2\pi^2}{U_1}E_1(\phi,U) \tag{33}$$

where $E_1(\phi,U)$ is a new name for an old expression, the by now familiar torus propagator:

$$E_1(\phi,U) = -\pi\ln\left|\frac{\vartheta_1(\phi/2\pi,U)}{\eta(U)}\right|^2 + 2\pi^2 U_2 a_1^2\ . \tag{34}$$

(Note that this is not *exactly* the expression we considered, but its T-dual, as explained in the appendix of [20]). Fortunately, (33) is an identity (p. 68 in [18]), as you can easily check using the explicit expressions and your previous experience with E_1, so the f corrections are consistent with the K corrections.

[11] "Kähler adapted vertex operators". This just means you compute with your usual vertex operators, but combine the results as we prescribe in the paper. One advantage of this is that integrating to the Kähler potential becomes trivial.

6.2 Toroidal orientifolds: $\mathcal{N} = 1$ sectors

For parallel branes, $\mathcal{N} = 1$ sectors do not contribute moduli dependence to $\mathcal{L}_{\text{eff}}$, so we could ignore them. For branes at angles, we introduce dependence on T, so we need to calculate the contributions due to $\mathcal{N} = 1$ sectors. Most but not all of these calculations have been performed as of the time of writing.

A. Corrections to f

There was a nice 2003 paper by Lüst and Stieberger [21] that found that if we discard divergent terms,

$$\int_0^\infty d\ell \, \frac{\vartheta_1'(ia)}{\vartheta_1(ia)} = -\frac{\pi}{2} \ln \left[e^{-2i\gamma a} \frac{\Gamma(1-ia)}{\Gamma(1+ia)} \right] \tag{35}$$

as you found. There is a curious connection to number theory (!), which you can find out yourself by starting with Appendix A of that paper (which is itself Exercise 6.13 in the number theory textbook [22]). To wet your appetite, a similar calculation was done in Riemann's 1859 paper "Über die Anzahl der Primzahlen unter einer gegebenen Grösse" [23], that founded analytic number theory and stated the Riemann Hypothesis!

The 2003 paper had a small mistake that was corrected in [24]. The final result is, taking into account all three T^2 factors in $T^2 \times T^2 \times T^2$ with individual angles θ^i,

$$\Delta = -\beta_a^{\mathcal{N}=1} \ln \left[\frac{\Gamma(\theta^1)\Gamma(\theta^2)\Gamma(1+\theta^3)}{\Gamma(1-\theta^1)\Gamma(1-\theta^2)\Gamma(-\theta^3)} \right] \tag{36}$$

Even though I am not giving any details of the derivation of (36), your calculation of (35) and the discussion of computing correlators in the 1st lecture, and the discussion of intersecting branes in the 2nd lecture, hopefully gives you some feeling for how (36) can arise.

B. Corrections to K

This was performed in 2007 by the Italian group involving our own Paolo Di Vecchia [25] and by the German group [26]. The calculation gives an intermediate result of the same form as above, e.g. (6.3) in [26]

$$K_{ab} \sim \ln[\mathbf{\Gamma}_{ab}] \tag{37}$$

where

$$\mathbf{\Gamma} = \left[\frac{\Gamma(\theta^1)\Gamma(\theta^2)\Gamma(1+\theta^3)}{\Gamma(\theta^1)\Gamma(\theta^2)\Gamma(-\theta^3)} \right] \tag{38}$$

and the indices a, b tell you between which brane stacks the angle is. The most recent paper about loop corrections is the two-point function calculation [27] that considers $K_{\phi\bar{\phi}}^{1-\text{loop}}$ for chiral fields, which is what we are most interested in. So to summarize, these calculations are not quite settled yet, but soon will be.

6.3 Why?

Just a short comment if you are lost in these detailed calculations: the point here was that if we want to consider some $\mathcal{L}_{\text{eff}}$ where string effects like α' corrections make a big difference, as in LVS, we should better understand all corrections that could potentially contribute with comparable numerical size. For example, the physical Yukawa couplings, which are of course very important for phenomenology, contain the Kähler metric of chiral matter fields (see the paper by Brignole at el I referenced in the previous lecture.)

6.4 Extrapolating to Calabi-Yau with fluxes

The above results for toroidal orientifolds were already technically challenging. For more phenomenologically interesting scenarios like LVS, we would need to generalize the calculations to curved backgrounds with fluxes. This has not been done. Instead, what I did with Michael Haack and Enrico Pajer was to *guess* the answer in section 3.1. of [7]. In one sample model with one $\tau_{\rm b}$ and one $\tau_{\rm s}$, we wrote for example

$$\Delta K_{g_{\rm s}}^{1-{\rm loop}} = \frac{\sqrt{\tau}_{\rm b}\mathcal{E}_{\rm b}^{(K)}(\phi,U)}{S_1\mathcal{V}} + \frac{\sqrt{\tau}_{\rm s}\mathcal{E}_{\rm s}^{(K)}(\phi,U)}{S_1\mathcal{V}} \tag{39}$$

for some *unknown* functions $\mathcal{E}_{\rm b}^{(K)}(\phi,U)$ and $\mathcal{E}_{\rm s}^{(K)}(\phi,U)$. We then found that for order one values of these functions, $\Delta K_{g_{\rm s}} \gg \Delta K_{\alpha'}$ numerically (the latter is given in eq. (16)). However, there are some miraculous cancellations, that still remain to be completely understood, when you compute the scalar potential V from (7) using the full corrected K. So, the original LVS results remain unchanged, for the case of Swiss cheese Calabi-Yaus. It seems that $\Delta K_{g_{\rm s}}$ does make a difference in some contexts, such as [28] and [15].

One can rightfully be skeptical of the guesswork involved in (39), though we give some arguments in [7] why (39) gives the right scaling with the Kähler moduli, which is the main issue here. Encouragingly, there are some ideas how to check this in more complicated cases. This is current work, as I now review.

6.5 Future: checking the extrapolation

According to [29], we can calculate at least some part of the open-string loop correction by a *supergravity tree-level* calculation, using open/closed duality. This has only been done for the *noncompact* Klebanov-Strassler Calabi-Yau manifold, where the explicit Calabi-Yau metric $\tilde{g}_{mn}$ is known. The calculation is still not easy; the integration over the 4-cycle on which the D7-branes are wrapped is quite involved. It actually uses AdS/CFT arguments! (I emphasize that when the smoke has cleared, there is no AdS/CFT duality involved, its only role was to give some very useful hints on *how to do* the calculation.) The result is

$$\tilde{A}(\phi) = A\left(\frac{\mu^P - \prod_{i=1}^4 \phi_i^{p_i}}{\mu^P}\right)^{1/N_{\rm D7}} \tag{40}$$

where μ is a constant, $P = \sum_i p_i$ and the D7-brane embedding is $f(\phi_0^i) = \prod_{i=1}^4((\phi_0^i)^{p_i} - \mu^P) = 0$. I expect this will be extended to other models and that it will be checked what part of the full string correction this reproduces in simpler models in the future. This development is interesting in itself, in my opinion. Comparing to eq. (29), we see a similar branch structure.

7 Another frontier: Quantum stabilization

Now that we have explored taking perturbative α' and $g_{\rm s}$ corrections into account, we could go crazy and forget about the nonperturbative $W_{\rm np}$ completely. This would in principle be nice since this is typically the murkiest part (cf. the discussion in section 3 about what A in $W_{\rm np}$ is, and whether $W_{\rm np}$ is generated for all cycles). If we could find vacua without $W_{\rm np}$, then in these vacua, the fact that the extra 6 dimensions are small is a *quantum string effect*, in the sense that both α' and $g_{\rm s}$ corrections play a crucial role.[12] I find this very interesting, since people have studied the phenomenology of supergravity models for quite a while, and each new such model is in essence yet another point particle model. String theory is so different that if such a crucial thing as stabilization depends on stringy effects, that means that these models should be qualitatively different from other supergravity models, and in principle allow us to probe string physics at least indirectly.

[12] Why this terminology? If it was only α', it could be a *classical* string effect, and if it was only $g_{\rm s}$, it could be a Yang-Mills effect in the limit $E^2\alpha' \to 0$, i.e. quantum but *non-string* effect. Similarly, $W_{\rm np}$ from gaugino condensation is a quantum effect, but a low-energy gauge theory quantum effect, not really dependent on any string (D-brane) representation of the Yang-Mills theory. So, something that arises when we consider both α' and $g_{\rm s}$ is a quantum string effect.

The bad news for quantum stabilization is that the large volume in LVS was achieved by having these exponentials in $W_{\rm np}$ around. If we don't have them, generically we might stabilize at small volume, unless there is some other mechanism. A first glance at this in the string context was [30].

The good news is that if (and you would have to read [30] first to understand this, but that paper is fortunately very short) we balance a factor of $g_{\rm s}$ against a factor of $\mathcal{V}^{1/3}$, it looks like quantum stabilization gets *better and better* for weak coupling, in the sense that we obtain minima at greater and greater volume $\mathcal{V}$ as we lower the string coupling $g_{\rm s}$. [13] This has not been explored much, partly because one would want to have more control over the loop corrections before one says too many detailed things of whether this really works or not, so right now the work in the previous subsection seems more urgent.

8 End: a quick sketch of origins

Many of these developments owe something at least indirectly to AdS/CFT:

$$\text{AdS/CFT} \;\Rightarrow \text{Randall-Sundrum} \Rightarrow \text{GKP} \Rightarrow \text{KKLT} \Rightarrow \text{ISS}. \tag{41}$$

so it's natural that many of these papers have the term "supergravity dual" in them. We also have

$$\text{AdS/CFT} \;\Rightarrow \text{Klebanov-Strassler} \Rightarrow \text{GKP} \tag{42}$$

And in cosmology, the circle was closed last year by ending up with Klebanov and Maldacena:

$$\text{KKLT} \Rightarrow \;\text{KKLMMT} \Rightarrow \text{BDKMMM [29]} \Rightarrow \text{BDKM[31]} \tag{43}$$

So, all you have to do now is to read all the following references :=)

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