Anomaly Mediated SUSY Breaking, JC 2010

References:
* L. Randall & R. Sundrum, "Out of this world ~~SUSY~~", hep-th/9810155
* J. Terning, "Modern SUSY, Dynamics and Duality"
* M. Dine & N. Seiberg, "Comments on Q effects in SUGRA", hep-th/0701023
* S.P. de Alwis, "On AMSB", 0801.0578
* S. Martin, "A SUSY primer", hep-ph/9709356

Plan:
* Basic idea of the setup in AMSB
* Slightly more technical walkthrough
  - Preliminaries:
    • Superfields & Supergravity
    • Spurions
    • Conformal compensator
  - Obtaining the mass contributions
  - The μ-problem
  - Phenomenology
* Discussion of recent confusion, based on
  - Dine & Seiberg, '07
  - de Alwis, '08

Basic Idea of AMSB

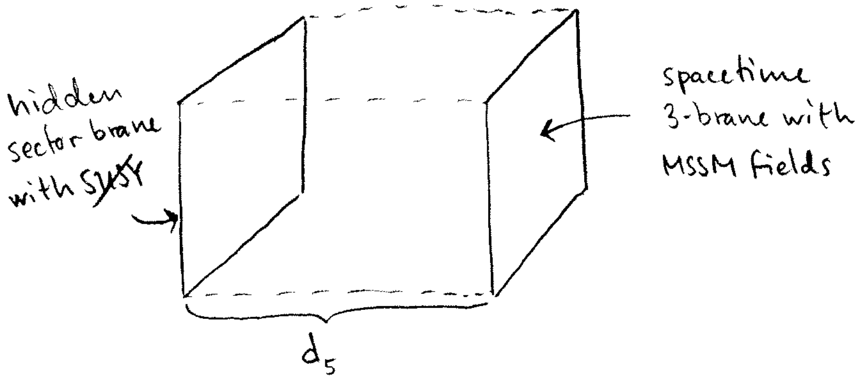
- Consider the hidden ~~SUSY~~ sector to be actual extra spacetime dimension

  * In such scenarios, the hidden sector is usually denoted a **sequestered sector**. In Randall-Sundrum proposal this is to mean truly hidden.

    - Derives from Latin sequestrare ~ "to give up for safe-keeping"

      Mainly used in legal context ~ "take temporary possession of property as security against legal claims"

- Simple illustration: Consider hidden sector to be a brane where SUSY is broken and ~~SUSY~~ effects get transmitted to our MSSM 3-brane through a connecting spatial dimension



Due to size, d_5, of extra dimension any massive hidden sector fields with mass $m_H > d_5^{-1}$ will have their interactions with MSSM fields Yukawa suppressed $\sim e^{-m_H d_5}$. In particular, there is a natural suppression of flavour changing neutral currents.

The ~~SUSY~~ effects are transmitted via gravitational effects. More precisely they can be seen as quantum anomalous scaling corrections producing gaugino masses at 1-loop and scalar masses^2 at 2-loop, as in gaugino mediated ~~SUSY~~. Sequestering guarantees these are the dominant contributions.

Randall-Sundrum, In a bit more detail

First some preliminaries

* Superfields & Supergravity

We introduce Grassmann variables $\theta, \bar{\theta}$, with $\{\theta_\alpha, \bar{\theta}_{\dot{\alpha}}\} = 0$

They obey the Berezin integration rules

$$\int d\theta = 0 \;, \quad \int d\theta\, \theta = 1 \;, \quad \int d^2\theta\, \theta^2 = 1 \qquad ; \; d^2\theta = -\tfrac{1}{4} d\theta^\alpha d\theta^\beta \epsilon_{\alpha\beta} \;, \; \epsilon_{12} = -1$$

$$\int d^4\theta\, \bar{\theta}^2 \theta^2 = \int d^2\theta\, d^2\bar{\theta}\, \bar{\theta}^2 \theta^2 = 1$$

$$\int d^2\theta\, (\chi\theta)(\psi\theta) = -\tfrac{1}{2} \chi\psi \quad \text{for arbitrary spinors } \chi, \psi$$

Then, using Superspace coordinates

$$y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$$

we can introduce chiral superfields

$$\Phi(y) = \underset{\uparrow\,\text{bosonic field}}{\varphi(x)} - i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi - \tfrac{1}{4}\theta^2\bar{\theta}^2 \Box\varphi + \sqrt{2}\,\theta \underset{\uparrow\,\text{fermionic field}}{\psi(x)} + \tfrac{i}{\sqrt{2}}\theta^2 \partial_\mu\psi\sigma^\mu\bar{\theta} + \theta^2 \overset{\text{auxilliary}\,\downarrow}{F(x)}$$

and vector superfields

$$V^a(y) = \theta\bar{\sigma}^\mu\bar{\theta} \underset{\text{gauge field}}{A^a_\mu} + \underbrace{i\theta^2\bar{\theta}\lambda^{\dagger a} - i\theta\bar{\theta}^2\lambda^a}_{\text{gaugino}} + \tfrac{1}{2}\theta^2\bar{\theta}^2 \underset{\text{auxilliary}}{D^a} \quad , \text{Wess-Zumino gauge}$$

This notation allows considerable simplifications in manipulating actions. As an example, the free kinetic lagrangian is simply given by

$$\int d^4\theta\, \Phi^\dagger\Phi = \partial^\mu\varphi^*\partial_\mu\varphi + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi + F^*F \underbrace{- \tfrac{1}{4}\partial^\mu(\varphi^*\partial_\mu\varphi + \partial_\mu\varphi^*\varphi) + \tfrac{i}{2}\partial_\mu(\psi^\dagger\bar{\sigma}^\mu\psi)}_{\text{total derivative}}$$

$$= \mathcal{L}_{free}$$

Also

$$\int d^2\theta\, W(\Phi) + h.c. = \mathcal{L}_{int} \;, \quad \int d^2\theta\, W^{a\,2}_\alpha + h.c. = \mathcal{L}_{SYM}$$

$W \sim$ holomorphic superpotential

$W^a_\alpha \sim$ field strength chiral superfield

* Conformal compensator & Spurion

Spurions: We can treat parameters of a theory as expectation values of some background fields. If we can enhance the symmetry of the theory by allowing the BG fields to transform under the symmetry, the parameters are called spurions since they allow for spurions enhanced symmetries that get spontaneously broken by the expectation values

- As an illustrative example: Consider the free kinetic Lagrangian with a wave renormalization factor

$$\int d^4\theta \, Z \bar{\Phi}^\dagger \Phi$$

We treat Z as a spurion by taking

$$Z = 1 + b\theta^2 + b^*\bar{\theta}^2 + c\theta^2\bar{\theta}^2$$

Then

$$\int d^4\theta \, Z \Phi^\dagger \Phi = \mathcal{L}_{free} + bF^*\varphi + b^*\varphi^* F + c\varphi^*\varphi$$

Integrating out F:s using the EoM we get

$$\int d^4\theta \, Z \Phi^\dagger \Phi = \underset{\substack{\uparrow \\ \text{no } F\text{:s}}}{\mathcal{L}_{free}} + \underbrace{(c - |b|^2)}_{\sim m^2_{soft} \text{ for scalars}} \varphi^* \varphi$$

Similarly we can generate the other soft ~~SUSY~~ terms by promoting yukawas, masses and the holomorphic gauge coupling (gaugino masses) to spurions.

Conformal compensator:

Consider ordinary Einstein-Hilbert gravity

$$S_{EH} = \frac{1}{2}\int d^4x \sqrt{-g}\, M_{pl}^2 R = \frac{1}{2}\int d^4x \sqrt{-g}\left[M_{pl}^2 R + \frac{1}{6}\partial_\mu M_{pl}\,\partial^\mu M_{pl}\right]$$

By promoting the Planck mass to a spurion field $M_{pl} \to \sigma$, we obtain the conformally invariant Brans-Dicke action

$$S_{BD} = \frac{1}{2}\int d^4x \sqrt{-g}\left[\sigma^2 R + \frac{1}{6}\partial_\mu\sigma\,\partial^\mu\sigma\right]$$

By now setting $\sigma = M_{pl}$ we break conformal invariance to Poincaré invariance and get back ordinary gravity.

Similarly in a SUSY version, we introduce a background chiral superfield

$$C = (\sigma, \chi, F_c)$$, the "conformal compensator"

and break conformal invariance by taking

$$C = (1, 0, F_c) = 1 + F_c\theta^2 \qquad (\text{or } M_0 + F_c\theta^2,\ M_0 \sim \text{D-dim Planck mass})$$

The conformal compensator is the spurion of the conformal symmetry.

Back to Randall-Sundrum setup

The general matter-coupled Lagrangian can be written (non-canonical form)

$$\int d^4\theta\, f(Q^\dagger, e^{-V} Q) C^\dagger C + \int d^2\theta \left(C^3 W(Q) + \tau(Q) W_\alpha^{a\,2} \right) + h.c$$

$$-\frac{1}{6} f(\tilde{q}^\dagger, \tilde{q}) \left[R + \text{gravitino \& vector auxilliary} \right]$$

Q - matter chiral superfields

$\tilde{q}$ - lowest scalar components of Q

τ - holomorphic gauge coupling function

f - Kähler function

$$f = -3M_{pl}^2\, e^{-K/3M_{pl}^2} \quad , K \sim \text{Kähler potential}$$

From the higher dimensional perspective, if we switch of gravity on the 4D MSSM brane ($g = \eta$, $C = 1$) then the hidden and visible sectors must decouple. This requires

$$f = -3M_{pl}^2 + f_{vis} + f_{hid}$$

$$W = W_{vis} + W_{hid}$$

$$\tau W_\alpha^{a\,2} = \tau_{vis} W_{\alpha\, vis}^{a\,2} + \tau_{hid} W_{\alpha\, hid}^{a\,2}$$

This form is what guarantees sequestering and appearance of $\cancel{\text{SUSY}}$ masses only at loop-level

It is a natural demand in the extra-dimensional scenario

Can be considered a Kähler "gauge fixing" (Pomarol & Ratazzi, '99)

Integrating out the hidden sector gives the 4D effective Lagrangian

$$\mathcal{L}_{eff} = \int d^4\theta\, Q^\dagger e^{-V} Q \frac{C^\dagger C}{M_0^2} + \int d^2\theta \frac{C^3}{M_0^3}\Big(\underset{\substack{\uparrow \\ \text{mass} \\ \text{parameter}}}{m_0} Q^2 + \underset{\substack{\uparrow \\ \text{superpotential} \\ \text{coupling} \\ \text{Non-renormalizable}}}{y_0} Q^3\Big) + \int d^2\theta\, \underset{\substack{\uparrow \\ \text{gauge effective} \\ \text{holomorphic} \\ \text{coupling}}}{g_0^{-2}} W_\alpha^{a\,2} + h.c. + \mathcal{O}\left(\frac{1}{M_0}\right)$$

A SUSY respecting field redefinition

$$\frac{CQ}{M_0} \to Q$$

brings this into the form

$$\mathcal{L}_{eff} = \int d^4\theta\, Q^\dagger e^{-V} Q + \int d^2\theta \left(m_0 C Q^2 + y_0 Q^3 + g_0^{-2} W_\alpha^{a\,2} + h.c.\right)$$

The MSSM gauge symmetries allows only the μ-term mass parameter, which we discuss later, so we set $m_0 = 0$ for now. Then clearly there is no trace of the compensator C in the classical Lagrangian, and hence no ~~SUSY~~ mass terms at tree level.

At the quantum level, the measure in the Wilsonian action is not invariant under the field redefinition so there are anomalous quantum corrections to the above that will involve C dependence. This will manifest in the RG running of couplings.

Cutoff dependence only enter in Kähler function f and holomorphic gauge coupling τ, so renormalizing down to a scale μ from Λ_{UV} we must have

$$\mathcal{L}_{eff} = \int d^4\theta\, \underset{\substack{\uparrow \\ \text{wave-function} \\ \text{renormalization}}}{Z\left(\frac{\mu M_0}{\Lambda_{UV}|C|}\right)} Q^\dagger e^{-V} Q + \int d^2\theta \left(y_0 Q^3 + \Big[\underset{\substack{\uparrow \\ \text{gauge function} \\ \text{renormalization}}}{g_0^{-2} + 2} \underset{\substack{\uparrow \\ \text{1-loop coefficient} \\ \text{in } \beta\text{-function}}}{b_0} \ln\left(\frac{\mu M_0}{\Lambda_{UV} C}\right)\Big] W_\alpha^{a\,2} + h.c.\right)$$

$\leftarrow$ Also R-symmetry considerations $\rightarrow$

<u>Note</u>: A non-zero m_0 will not enter any soft masses determined after renormalization, it will only alter the renormalized couplings values in terms of the UV values of the couplings which we can't measure anyhow. We only make measurements at the electro-weak scale

$\cancel{SUSY}$ in the Randall-Sundrum setup

* Consider the renormalized gauge holomorphic function

$$\tau_R = g_0^{-2} + 2b_0 \ln\left(\frac{\mu M_0}{\Lambda_{uv} C}\right) = g_0^{-2} - 2b_0 \ln\left(\frac{C}{M_0}\right) + 2b_0 \ln\left(\frac{\mu}{\Lambda_{uv}}\right)$$

Clearly, even without renormalization $(\mu \sim \Lambda_{uv})$ there is a shift

$$g_0^{-2} \to g_0^{-2} - 2b_0 \ln\left(\frac{C}{M_0}\right)$$

The extra contribution is due to the quantum Super-Weyl anomaly

Expanding the logarithm, the F-term of C will generate a gaugino mass term (c.f. comment in connection with spurions) after θ-integration

$$m_\lambda = -b_0 g^2 \frac{F_C}{M_0} \quad , \quad g \sim \text{gauge coupling} \quad , \quad F_C \sim m_{3/2} \text{ gravitino mass}$$

Thus, quite generally there is a 1-loop appearance of a $\cancel{SUSY}$ gaugino mass.

* Consider next the wave function renormalization factor

$$Z\left(\frac{\mu M_0}{\Lambda_{uv}|C|}\right) = \exp\left[\ln Z\left(\frac{\mu M_0}{\Lambda_{uv}|C|}\right)\right]$$

Expanding the logarithm

$$\ln Z\left(\frac{\mu M_0}{\Lambda_{uv}|C|}\right) = \ln Z\left(\frac{\mu}{\Lambda_{uv}}\right) - \frac{1}{2}\gamma\left(\frac{F_C}{M_0}\theta^2 + \text{h.c.}\right) + \frac{1}{4}\frac{|F_C|^2}{M_0^2}\theta^2\bar\theta^2\left(\frac{\partial\gamma}{\partial g}\beta_g + \frac{\partial\gamma}{\partial y}\beta_y\right)$$

in terms of RG functions

$$\gamma \equiv \frac{\partial \ln Z}{\partial \ln\mu} \quad ; \quad \beta_g \equiv \frac{\partial g}{\partial \ln\mu} \quad ; \quad \beta_y \equiv \frac{\partial y}{\partial \ln\mu}$$

and doing a field redefinition

$$\exp\left[\ln Z\left(\frac{\mu}{\Lambda_{uv}}\right) - \frac{1}{2}\gamma\left(\frac{F_C}{M_0}\theta^2 + \text{h.c.}\right)\right] Q \to Q$$

we see that only the last term in the log-expansion remains in the Lagrangian (ignoring further loop-supressed anomaly contributions)

We then expand the exponential and effectively have found that

$$Z\left(\frac{\mu M_0}{\Lambda_{uv} C}\right) = 1 + \frac{1}{4}\frac{|F_c|^2}{M_0^2}\theta^2\bar{\theta}^2\left(\frac{\partial\gamma}{\partial g}\beta_g + \frac{\partial\gamma}{\partial y}\beta_y\right)$$

After $\theta,\bar{\theta}$-integrations this generates scalar masses (c.f. section on spurions)

$$m_{\tilde{q}}^2 = -\frac{1}{4}\frac{|F_c|^2}{M_0^2}\left(\frac{\partial\gamma}{\partial g}\beta_g + \frac{\partial\gamma}{\partial y}\beta_y\right)$$

This is a 2-loop contribution so we need the RG functions to 1-loop

$$\gamma = c_0 g^2 + d_0 y^2 \;;\; \beta_g = -b_0 g^3 \;;\; \beta_y = e_0 y^3 + f_0 y g^3$$

In terms of these then

$$m_{\tilde{q}}^2 = \frac{1}{2}\left[c_0 b_0 g^4 - d_0 y^2\left(e_0 y^2 + f_0 g^2\right)\right]\frac{|F_c|^2}{M_0^2}$$

The 1^{st} term is positive for asymptotically free gauge theories but negative otherwise. Thus, since $SU(2)_L$ and $U(1)_Y$ are not asymptotically free the above expression implies that the MSSM sleptons are tachyonic (the squarks are ok though since $SU(3)_c$ is asymptotically free).

This is ofcourse a major drawback but can be remedied in various ways, eg

- Introducing additional bulk fields coupling MSSM leptons to hidden sector fields
- Introduce new Higgses with large Yukawas
- Introduce a heavy ~~SUSY~~ threshold with light singlets

* Finally, expanding the superpotential after the field redefinition also gives scalar interactions with trilinear coupling proportional to yukawa coupling

$$A_{ijk} = \frac{1}{2}\left(\gamma_i + \gamma_j + \gamma_k\right) y_{ijk}\,\frac{F_c}{M_0}$$

in analogy with gauge mediation scenario. Here, the regulator $\frac{\Lambda_{uv} C}{M_0}$ acts as a messenger in that language.

The μ-problem

We recall that in order for the MSSM to produce a phenomenologically viable mass spectrum, ie electroweak symmetry breaking with finite vev; we need μ and μB terms in the super- and scalar potentials

$$W \supset \mu H_u H_d \quad , \quad V \supset \mu B \, H_u H_d$$

and require

$$\mu^2 \sim \mu B \quad , \quad \mu \sim \mathcal{O}(m_{soft}) \sim \mathcal{O}(m_W)$$

Now, in principle μ and μB are unrelated so there is generally nothing forcing them to be of the same order

Also, the μ-term is supersymmetric so there is nothing obvious relating it to the electroweak scale

This is the μ-problem in a nut-shell

The μ-problem is not naturally resolved within the general setting of the Randall-Sundrum scenario, but can be resolved in principle within specific scenarios (see e.g. Terning p. 267)

Phenomenology

Here we restate the MSSM mass spectrum as given in Randall & Sundrum's paper, where it is assumed there will be some extra contributions rendering the scalar masses non-tachyonic

scalars
$$\begin{cases} m^2_{\text{sleptons}} = -1{,}3 \cdot 10^{-5} \dfrac{|F_c|^2}{M_D^2} + m^2_{\text{bulk},\ell} \\[2ex] m^2_{\text{squarks}} = 5{,}5 \cdot 10^{-4} \dfrac{|F_c|^2}{M_D^2} + m^2_{\text{bulk},H} \\[2ex] m^2_H = -1{,}3 \cdot 10^{-5} \dfrac{|F_c|^2}{M_D^2} + m^2_{\text{bulk},H} \end{cases}$$

gauginos
$$\begin{cases} m^2_{\text{gluino}} = 6{,}1 \cdot 10^{-4} \dfrac{|F_c|^2}{M_D^2} \\[2ex] m^2_{\text{wino/zino}} = 6{,}4 \cdot 10^{-6} \dfrac{|F_c|^2}{M_D^2} \\[2ex] m^2_{\text{bino}} = 7{,}0 \cdot 10^{-5} \dfrac{|F_c|^2}{M_D^2} \end{cases}$$

- The gaugino mass ratios are fixed

$$m_{\text{gluino}} : m_{\text{wino/zino}} : m_{\text{bino}} = 3 : 0{,}3 : 1$$

- The wino/zino are the lightest gauginos and depending on $m_{\text{bulk},\ell}$ they might also be the LSP.

Confusing aspects in AMSB

Some aspects of the AMSB arguments are confusing

* The anomaly responsible for the ~~SUSY~~ mass terms is an anomaly of a symmetry which classically does not exist!
  What does this mean?

* The formalism used here depended on the introduction of a spurion, which should be non-dynamical and simply be used as a formal tool for writing the action in a convenient way.
  Yet, at the quantum level the spurion reappeared and was manifested in physical quantities such as masses.
  Is this really a correct treatment?

Discussion in Dine & Seiberg, '07; arXiv: 0701023

* Agree with results of Randall & Sundrum and others. They maintain that these derivations are formally correct.

* Disagree with physical interpretation of the effect as due to an anomaly; "anomaly is a lack of symmetry, which cannot be restored by local counterterms".
  They argue instead that these terms are needed to preserve SUSY

* Illustrate similarities to these terms appearing in ordinary QFT, eg QED
  Work out several examples to demonstrate that such terms are independent of any SUGRA formulation and can even be obtained in global SUSY.
  In particular, analyzing the model in a Higgs phase they demonstrate the necessity of these terms to preserve SUSY. The appearance in the Wilsonian action of terms violating local SUSY is needed to compensate lack of invariance of the measure of light fields.

Discussion in de Alwis, '08; arXiv: 0801.0578

* Disagree with results of Randall & Sundrum and others.

  - Claims to correct the gaugino mass term due to the Weyl anomaly
  - Argues that the Dine & Seiberg gaugino mass term is an independent contribution which should add to the Weyl anomaly term
  - Argues that there are no anomaly mediated contributions to scalar masses and that these derivations are logically flawed.

  Agrees with the existence of Dine & Seiberg scalar mass terms but again argues these are distinct, but contrary to Dine & Seiberg he argues that they are always non-tachyonic